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Cosmographic nature of the early universe from extra dimensional perspective

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ABSTRACT

In this study, making use of the $f(R)$ theory of gravity in a D-dimensional Kaluza-Klein type framework, we discuss whether we need to assume additional dimensions in order to explain two stages (inflation and deceleration phases) of the universe. To reach this goal, we start with a higher dimensional form of the Friedmann equation written in the $f(R)$ gravity. For the inflation era occurring after a beginning vacuum phase of the universe, we conclude that a 10-dimensional spacetime model is needed as predicted in the string theory. However, after using a fixed field condition, it is observed that additional dimensions are compactified on the standard four spacetime dimensions in the deceleration phase. Moreover, the additional dimensions may be responsible for the dynamics of the dark energy dominated era, that's why we can state that the source of dark energy may be the extra dimensions.

1. Introduction

According to the string theory [1], we need additional dimensions in order to describe the general structure of the universe or construct a unified form for the fundamental forces we observe in nature. The Kaluza-Klein idea [2,3], which is known as the starting point of the string theory, is one of the significant attempts introduced to obtain a unified theory. The inability of the direct observation of fundamental particles of dark energy in the four dimensional framework has strengthened [4] the belief of the existence of the extra dimensions which are assumed to be compactified in every point at spatial scale of the four-dimensional spacetime. The dynamical behavior of the additional dimensions with time occurs in an extremely small area, therefore we could not observe it yet.

In literature, there is a great interest in cosmological investigations of the extra dimensions [4–12]. Most of the previous papers given in literature are generally based on exact solutions of the higher dimensional field equations and compactification of the extra dimensions. Moreover, it is generally accepted that the observed universe emerges from higher dimensions. For instance, we must assume that the compactification of extra dimensions happened before the inflation phase in order to describe the early inflationary era, which means the compactification condition is considered during birth of the universe. However, in the present study, we show that the extra dimensions may have been created during two regimes (quintessential and dust-like) of the inflation phase after a vacuum state of the universe. Mohammedi [9], after deriving the four-dimensional standard Friedmann equations from the higher dimensional Kaluza-Klein framework, constructed a cosmic scale factor for the extra dimensional contributions, i.e. $b(t) = a(t)^{-n}$ where $a(t)$ denotes the observed scale parameter and n must be positive to provide the compactification condition. Subsequently, Andrew et al. [10] studied the late time dynamical behavior of the universe by considering a higher dimensional Gauss-Bonnet term with the choice of $b(t) = a(t)^{-n}$. On the other hand, in Ref. [13], after assuming a higher dimensional $f(R)$-gravity scenario including the choice of $f(R) = R + R^n$ with a cosmological constant and performing a conformal transformation, an inflationary scalar potential was defined by making use of the compactification condition, and it was shown that the model leads to the Starobinsky inflation [14]. It is known that, after creation of the matter during inflation, the universe enters a deceleration regime and then it returns to an accelerated phase (the quintessence epoch) again. In this sense, the known formation of the existence of the extra dimensions may yield more information at the point of understanding dark energy puzzle or the large-scale structure of the universe as a whole. The super inflation mechanism together with the standard four-dimensional Starobinsky model in-

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dicates a super accelerated inflation period after a vacuum state of the universe [15–17]. From this point of view, in this study, the higher dimensional perspective with the Starobinsky model is directly examined to identify the super accelerated period. To reach this aim, the higher dimensional Friedmann picture is divided into two parts: the first one includes the standard four spacetime dimensions and the latter one is a part including the extra spatial dimensions. After guarantying the expansion phase of the universe in the standard four dimensional part of the spacetime via making use of the standard Friedmann-Lemaitre-Robertson-Walker (FLRW) cosmology and the super accelerated solutions, the existence of the extra dimensions is discussed in this perspective.

The layout of this paper is as follows. In the next section, the extra dimensional cosmology in the $f(R)$-gravity is introduced. In the third section, the existence of extra dimensions are discussed by considering the early inflationary regime, super accelerated phase and the deceleration epoch together with the Starobinsky model. Finally, in the fourth section, we summarize our results.

2. Preliminaries: cosmological scenario

In this section, we start with $D = 4 + d$ dimensional action of the $f(R)$-gravity which is written as [18]

$$S = \frac{1}{2k_D^2}\int d^D x\sqrt{-g_D}\, f(R) + \mathfrak{S}_m^{(D)}, \tag{1}$$

where k_D^2 is D-dimensional coupling constant, $\mathfrak{S}_m^{(D)}$ shows matter part of the total action and $f(R)$ denotes an arbitrary function of the Ricci scalar constructed by using D-dimensional metric tensor g_{AB} with $(A, B, ... = 0, 1, .., d+3)$ where d indicates the number of the extra dimensions. Once the case $d = 0$ is considered, the higher dimensional metric tensor is reduced to the conventional four dimensional one. Varying the above action with respect to the higher dimensional metric tensor yields the following field equations [18]

$$R_{AB} f_R - \frac{1}{2} f(R) g_{AB} - \left(\nabla_A\nabla_B - g_{AB}\nabla^2\right) f_R = k_D^2 T_{AB}. \tag{2}$$

Here, we defined that $f_R = \frac{\partial f}{\partial R}$. Additionally, in the above equation, ∇_A and $\nabla^2 \equiv \nabla^A\nabla_A$ are the covariant derivative and the D'alembertian operator in D-dimensional framework, respectively. The matter content of the universe is assumed to be a perfect fluid that is described by the energy-momentum tensor [9,10]

$$T_{AB} = diag\left[-\rho(t), p(t), p(t), p(t), p_d(t), ..., p_d(t)\right] \tag{3}$$

where $p_d(t)$ shows the pressure along all the additional dimensions.

A generalized form of the FLRW metric [9,10] which includes extra dimensions is defined as given below

$$\begin{aligned} ds^2 = &-dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] \\ &+ b^2(t)\gamma_{mn} dy^m dy^n \end{aligned} \tag{4}$$

where γ_{mn} and $b(t)$ show the metric tensor and the scale factor for the extra dimensional part of the universe, respectively. Note that, the indices of the metric tensor γ_{mn} take the values $(4, 5, ..., d)$. In addition to this, K shows the spatial curvature parameter for the flat $(K = 0)$, closed $(K = -1)$ and open $(K = +1)$ universe types. Some astrophysical observations such as SNe-Ia [19], WMAP [20–22], SDSS [23], X-ray [24] and Planck-results [25–27] have strongly indicated a spatially flat geometry of the universe. Consequently, in further calculations, we study on a manifold with the flat curvature case $K = 0$ for visible universe dimensions. Therefore, considering the spatial flat case may also lead to the assumption of the isotropy and homogeneity for the extra dimensions. Additionally, assuming compactification of the extra dimensions in every point of the visible standard four-dimensional spacetime may indicate a flat form due to the recent observational data of the visible universe. Hence, the Riemann tensor of the extra dimensions is chosen as the maximally symmetric, i.e. $R_{abcd} = \tilde{K}(\gamma_{ac}\gamma_{bd} - \gamma_{ad}\gamma_{bc})$, where the curvature parameter of the extra dimensions $\tilde{K}$ is assumed to be equal to zero. Thence, assuming $\tilde{K} = 0$ leads to a flat manifold for the extra dimensions.

After making use of the field equation (2) with the line-element (4), the higher dimensional Friedmann equations (HDFE) can be obtained as

$$-\frac{3\ddot{a}}{a} f_R + \frac{1}{2} f(R) + \frac{3\dot{a}}{a}\dot{f}_R - \frac{d\ddot{b}}{b} f_R + \frac{d\dot{b}}{b}\dot{f}_R = k_D^2\rho, \tag{5}$$

$$\begin{aligned} &\left(\frac{\ddot{a}}{a} + \frac{2\dot{a}^2}{a^2}\right) f_R - \frac{1}{2} f(R) - \frac{2\dot{a}}{a}\dot{f}_R - \ddot{f}_R \\ &+ \frac{d\dot{a}\dot{b}}{ab} f_R - \frac{d\dot{b}}{b}\dot{f}_R = k_D^2 p, \end{aligned} \tag{6}$$

$$\begin{aligned} &\left(\frac{\ddot{b}}{b} + \frac{(d-1)\dot{b}^2}{b^2} + \frac{3\dot{a}\dot{b}}{ab}\right) f_R - \frac{1}{2} f(R) \\ &+ (1-d)\frac{\dot{b}}{b}\dot{f}_R - \ddot{f}_R - \frac{3\dot{a}}{a}\dot{f}_R = k_D^2 p_d. \end{aligned} \tag{7}$$

Note that, here, the dot implies a time derivation. It is important to mention here that the quantities $f(R)$ and f_R can be computed in terms of the scale factors a and b.

Moreover, we find the following relation for the higher dimensional Ricci scalar by using the surviving components of the Ricci tensor

$$R \Longrightarrow R + R^{(d)} = 6\dot{H} + 12H^2 + \frac{2d\ddot{b}}{b} + 6dH\frac{\dot{b}}{b} + d(d-1)\frac{\dot{b}^2}{b^2}, \tag{8}$$

where $H = \frac{\dot{a}}{a}$ is the cosmic Hubble expansion parameter.

On the other hand, the conservation equation or the Friedmann equations (5), (6) and (7) leads to [9]

$$\dot{\rho} + 3H(\rho + p) = -d\frac{\dot{b}}{b}(\rho + p_d). \tag{9}$$

The HDFE and the above conservation relation can be rewritten in a more elegant form [18,28–30]. On this purpose, one can find

$$3H^2 = k_D^2\rho_{eff}, \tag{10}$$

$$2\dot{H} + 3H^2 = -k_D^2 p_{eff}, \tag{11}$$

and

$$\dot{\rho}_{eff} + 3H(\rho_{eff} + p_{eff}) = 0, \tag{12}$$

with the help of the effective energy density ρ_{eff} and pressure p_{eff} which are defined as

$$\begin{aligned} \rho_{eff} = \frac{1}{f_R}\Big[&\rho + \frac{1}{k_D^2}(3\dot{H} f_R + 6H^2 f_R - \frac{1}{2} f(R) - 3H\dot{f}_R \\ &+ \frac{d\ddot{b}}{b} f_R - \frac{d\dot{b}}{b}\dot{f}_R)\Big], \end{aligned} \tag{13}$$

$$p_{eff} = \frac{1}{f_R}\Bigg[p - p_d + \frac{1}{k_D^2}\left(\left\{\frac{\ddot{b}}{b} + \frac{(d-1)\dot{b}^2}{b^2} + (3-d)H\frac{\dot{b}}{b} - 3\dot{H} - 6H^2\right\} f_R + (1+2H)\dot{f}_R\right)\Bigg]. \tag{14}$$

3. Cosmography of the early universe

At this step, we are in a position to discuss the early inflation and the deceleration phases of the universe in the extra dimensional $f(R)$ theory of gravity. In order to identify the early inflation and the deceleration regimes of the universe in a unified relation [15–17,31], one can construct a very useful quantity defined in the following form

$$A = \ddot{f}_R - (1+3\omega)\dot{H} f_R + (2+3\omega)H\dot{f}_R + (1+\omega)\left(\frac{f(R)}{2} - 3H^2 f_R\right) + d\left[\frac{\dot{b}}{b}(1+\omega)\dot{f}_R - \frac{\dot{a}\dot{b}}{ab}\dot{f}_R - \frac{\ddot{b}}{b}\omega f_R\right]. \tag{15}$$

We must have $A=0$ to get $\omega = \omega_{eff} = -1 - \frac{2\dot{H}}{3H^2}$ for the equation-of-state (EoS) parameters. Therefore, this condition states that the visible universe can be a reference point for our discussion. The above relation may also be considered as another form of the Friedmann equations including the effective EoS parameter. It can be said that the historical expansion of the universe can be described by making use of this quantity for all $f(R)$-gravity definitions. The geometric $f(R) = R + \alpha R^2$ model describes well [14] the early inflationary regime of the universe. In addition to this, it can be concluded that the super inflation mechanism is discussed in a different manner by considering the equation (15) directly. In one respect, it implies a unification of the exponential and the power-law solutions of the cosmic scale factor. According to the super inflation mechanism, the universe starts its evolution with a vacuum state (the exponential solution) and then it enters a super accelerated phase (the power-law solution). At the end of the inflation phase, the creation of a radiation-like fluid, which is described by the EoS parameter $\omega \sim \frac{1}{3}$, is observed. After that, the universe smoothly enters the deceleration epoch due to the creation of matter during inflation. Therefore, one can extend the general theory of relativity by assuming $f(R) = R + \alpha R^2$. Naturally, the above result describes both precursor phases.

Taking the limiting case $d \to 0$ leads to the four dimensional framework. For the simplicity and further discussions, let's discuss this limiting case as a starting point. Hence, we get

$$A = \ddot{f}_R - \dot{H} f_R(1+3\omega) + H\dot{f}_R(2+3\omega) + \left[\frac{f(R)}{2} - 3H^2 f_R\right](1+\omega). \tag{16}$$

Assuming a quasi-de Sitter vacuum state including the cases $\omega \cong -1$, $\dot{H} \sim 0$ and $R \cong 12H^2$ in the equation above leads to the relation $2\dot{H}(12\dot{H}^2 f_{RR} - f_R) = 0$. Consequently, we find $f(R) \sim \alpha R^2$. On the other hand, making use of the first Friedmann equation (5) with the $d \to 0$ condition produces an exponential scale factor $a = a_0 e^{\sqrt{\frac{k^2\rho}{3}}t}$. Additionally, if the geometric αR^2 term is taken into account as a fixed field description of the inflation phase, the universe can enter a super inflationary period defined by the power-law form of the scale factor, i.e. $a = a_0 t^h$. Consequently, in this case, one can write $3\alpha(2h-1)[4-3h(1+\omega)]R^2 = 0$, which produces the super accelerated solution [31,32] $h = \frac{4}{3(1+\omega)}$. It can be concluded [17] that the range $h > 2$ implies a quintessential expansion with the range $-1 < \omega < -\frac{1}{3}$. Next, the range $1 < h < 2$ indicates the creation phase of the baryonic matter defined by the interval $-\frac{1}{3} < \omega < \frac{1}{3}$. It is important to mention here that the reheating phase is also observed in the range $-\frac{1}{3} < \omega < \frac{1}{3}$.

Now, focusing on the expansion perspective of the observed four spacetime dimensions, we discuss the existence of the extra dimensions. Thus, the standard four dimensional cosmology will be a reference point to us for the investigation.

Mohammedi [9] defined an alternative formulation for the effective pressure to hold the conservation of ordinary matter described in the equation (9). The author wrote that [9]

$$\dot{\rho} + 3H(\rho + \widetilde{p}) = 0, \tag{17}$$

where

$$\widetilde{p} = p - \frac{nd}{3}(\rho + p_d) \tag{18}$$

is an effective pressure description. In the equation (14), we gave another definition of the effective pressure. It is seen that one can reduce the extra-dimensional conservation relation (14) or (17) to the standard one without taking the limit $d \to 0$. We just need to take $\frac{dn}{3}(\rho + p_d) = 0$. After focusing on the metric (4) with the help of Mohammedi's description [9] $b = a^{-n}$, we can conclude that, if $n > 0$, the scale factor b continuously shrinks in a very small volume, which is known as the Kaluza-Klein space, when the observed scale factor a grows rapidly. Therefore, it leads to the standard four dimensional universe at the late-time phase. This is the main idea of the dynamical compactification. On the other hand, if we have the case $n < 0$, the model yields a visible D-dimensional universe. For instance, the case $n = -1$ with $d = 1$ leads to the classical five-dimensional Kaluza-Klein metric [33]

$$ds^2 = -dt^2 + a^2(t)\left[dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2\right]. \tag{19}$$

In order to determine the number of the additional dimensions in a simple way, we can write $\dot{b} = -n\frac{\dot{a}}{a^{n+1}}$. Thence, equations (9) or (17) can be rewritten in the following form

$$\dot{\rho} + 3H(\rho + p) = \frac{nd(\rho + p_d)}{b}\frac{\dot{a}}{a^{n+1}}. \tag{20}$$

Here, assuming the case $n = -1$ as a fixed point for an instance, we get the EoS $p_d = -\rho$ which provides conservation of the ordinary matter. Also, it can be concluded that the master equation (15) yields the same result at the point $n = -1$ when $A = 0$. As a result, we can state that the equation (15) can be taken into account as a D-dimensional energy relation.

3.1. Inflation era

Here, we consider the super inflation mechanism to investigate the inflation phase of the early universe. Recall that, in this mechanism [15–17], the universe starts with a vacuum state and then enters a super accelerated period which constitutes a quintessential field and a matter creation regime. For the vacuum state, the higher dimensional Ricci scalar (8) is read as the form $R = (12 + dn^2 - 6dn + d^2n^2)H_0^2$. Due to higher curvatures in the early universe, we may assume the quadratic case $f(R) = \alpha R^2$. Consequently, using the master equation (15), we get

$$[nd(nd+n-6)+12]\left[(1+n)-\frac{\dot{H}_0}{H_0^2}\right]=0. \quad (21)$$

From this point of view, it is calculated, in the limiting case $d=0$, that $(1+n)=\frac{\dot{H}_0}{H_0^2}$ which provides the quasi-de Sitter expansion around the point $n\sim-1$ with $\dot{H}_0\sim 0$ and $R=12H_0^2$. But, we assume $d\neq 0$ in our investigation. In this way, Eq. (21) leads to the following solutions

$$n=\frac{3d\pm\sqrt{-3d(d+4)}}{d(d+1)}, \quad (22)$$

and

$$n=\frac{\dot{H}_0}{H_0^2}-1. \quad (23)$$

The equation (22) with the case $d>0$ yields that n is a complex number. The second solution (23) hereby should be considered in order to reach physically meaningful discussions. Also, the latter case is the same with the limiting case $d=0$ for the quasi-de Sitter type expansion. The accelerated quasi-de Sitter expansion $\dot{H}_0\sim 0$ yields $n\sim-1$. Therefore, we concluded that the universe emerged from the standard four spacetime dimensions although we assume that the structure of the universe is constituted from D spacetime dimensions. In the previous discussions, we have mentioned that the case $n=-1$ provides the conservation of the ordinary matter with $p_d=-\rho$. Therefore, the following relation is computed

$$\frac{\dot{H}_0}{H_0^2}\left[n(d-1)-3-2n\right]=3(1+n). \quad (24)$$

We see that the quasi-de Sitter expansion case $\dot{H}_0\sim 0$ gives the status $n=-1$ which implies that the universe expands around this point during the vacuum inflation era.

3.2. Super acceleration phase

After the vacuum state, in which the accelerated quasi-de Sitter expansion is governed by αR^2-term, the universe can be in the super acceleration period due to the existence of this quadratic term in the model. In other words, the inflation phase of the universe can be characterized by this term as long as higher curvatures of the early universe is conserved. In this sense, making use of the relation $b=a^{-n}$ with the power-law definition of the observed scale factor gives the following description of the Ricci scalar,

$$R=t^{-2}\left(2hnd+12h^2-6h+n^2h^2d+n^2h^2d^2-6nh^2d\right). \quad (25)$$

On the other hand, considering the relation $A=0$ with the quadratic term αR^2, we get

$$3[4-3h(1+\omega)] = hnd\left[2n\omega h-2h-2\omega-4-\frac{1+\omega}{2}\left(h[n-6+nd]+2\right)\right]. \quad (26)$$

Since the universe has a standard four-dimensional spacetime structure in the vacuum state, an observer, in the super accelerated period with $h=\frac{4}{3(1+\omega)}$, reads the equation (26) as follows

$$d=-\frac{11+12\omega+9\omega^2+2n(1+\omega)}{2n(1+\omega)} \quad (27)$$

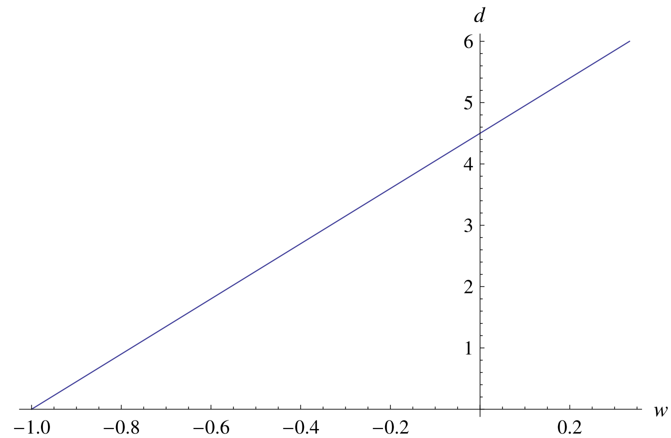

Fig. 1. The evolution of number of the extra dimensions versus the effective EoS parameter with $n=-1$.

where $\omega\neq-1$. The super acceleration period $-1<\omega<\frac{1}{3}$ can be divided into two territories described by $-1<\omega<-\frac{1}{3}$ and $-\frac{1}{3}<\omega<\frac{1}{3}$ intervals and we have the case $n<0$ with $d>0$ in this period. Hence, it can be found by using $-\frac{11+12\omega+9\omega^2+2n(1+\omega)}{2n(1+\omega)}>0$ that we have $-1<\omega<\frac{1}{3}$ if $n<0$. Furthermore, from the equation $p_d=-\rho$, we can obtain the same result with (27). Thus, we can see that the early accelerated expansion of the universe occurs around $n=-1$. The analysis of $n=-1$ condition in the conservation equation (20) should be reconsidered for all the cases. For the vacuum state solution, in which we have the point $n=-1$, one can see that we get $\frac{\dot{a}}{a^{1+n}}=\dot{a}$ and the right-hand-side of the equation (9) is transformed into the form of $-d(\rho+p_d)H_0$. Also, due to the point $n=-1$, we have the EoS $p_d=-\rho$ which provides the conservation of matter. Nevertheless, the power-law solution gives $\frac{\dot{a}}{a^{1+n}}=\frac{ht^{h-1}}{t^{h(n+1)}}$. Now, taking $n=-1$ transforms the right-hand-side of the equation (9) into the form of $-\frac{hd}{t}(\rho+p_d)$ which means we must have $p_d=-\rho$ in order to reach the conserved matter case. Hence, when the value $n=-1$ is fixed in the equation (27) for $\omega\sim\frac{1}{3}$ which specifies the end of the inflation regime, the number of additional dimensions is calculated as $d=6$ which is the predicted value of the string theory. As a conclusion, we reach a 10 dimensional form of the universe which was created after the vacuum state. In Fig. 1, we illustrated the $d\sim\omega$ relation. In the range $0<d\leq 6$, the EoS parameter evolves as $-1<\omega\leq\frac{1}{3}$, where $\omega\cong\frac{1}{3}$ shows the end of the inflation stage. In the vacuum state, the number of the extra dimensions becomes equal to zero, thus the case yields the standard four dimensional framework. Moreover, we get $d=\frac{9}{2}(1+\omega)$ form the equation (27). Hence, this figure must be read as $\omega=-\frac{5}{9}$ for $d=2$, $\omega=-\frac{1}{9}$ for $d=3$ and $\omega=-\frac{1}{9}$ for $d=4$, etc.

3.3. Deceleration regime

Here, we focus on the Einstein proposal to investigate the numbers of the extra dimensions created in the inflation stage. The creation of the matter during the inflation phase can provide that the universe smoothly enters into the standard radiation era. For the deceleration region of the universe, D-dimensional Einstein model $f(R)=R+R^{(d)}$ should be taken into account because it includes also the standard Einstein term R. For this case, we can write

$$2\dot{H}+3H^2(1+\omega) = -d\left[\frac{\dot{b}}{b}+\frac{\dot{a}\dot{b}}{ab}(2+3\omega)+\frac{(1+\omega)(d-1)\dot{b}^2}{2b^2}\right]. \quad (28)$$

Considering the case $\omega=-1-\frac{2\dot{H}}{3H^2}$ with $d\neq 0$, the above equation gives

$$\frac{\dot{b}}{b}+\frac{\dot{a}\dot{b}}{ab}(2+3\omega)+\frac{(1+\omega)(d-1)\dot{b}^2}{2b^2}=0. \quad (29)$$

At this step, after assuming [9] $b=a^{-n}$, we get

$$H^2(4+6\omega-2n-n[1+\omega][d-1])=-2\dot{H}. \quad (30)$$

Now, focusing on the non-phantom power law solution $a=a_0t^h$, one can find that

$$h=\frac{2}{(4+6\omega-2n-n[1+\omega][d-1])}=\frac{2}{3(1+\omega)}. \quad (31)$$

It should be emphasized here that the above expression can also be obtained from the EoS relation $p_d=-\rho$ which means it is a fixed case for the inflation and the deceleration stages. So, we get

$$n=\frac{1+3\omega}{1+d+\omega(d-1)}. \quad (32)$$

Under the compactification condition, where n must be bigger than zero and $d\geq 1$, we find that $\omega>-\frac{1}{3}$. This conclusion indicates the deceleration phase of the universe precisely. The equation (32) is also known as the standard four-dimensional FLRW case which means the 10-dimensional universe evolved to the standard four-dimensional structure after the inflation phase.

4. Closing remarks

In this study, for the two regimes of the universe (inflation and deceleration), we have investigated the existence of the extra dimensions in context of the Kaluza-Klein framework with the higher dimensional $f(R)$ gravity. On this purpose, we start with focusing on the dynamical nature of the vacuum state which can be the beginning expansion phase of our universe and which arises in usual four spacetime dimensions. In the direction of this motivation, besides the Einstein term describing deceleration regime of the universe, we consider a D-dimensional model involving the standard Starobinsky proposal.

Since, we have found the same results both in the inflation and the deceleration regimes, the relation $-\rho=p_d$ emerged as a fixed field for both of the regimes mentioned above. It is generally known that the accelerated quasi-de Sitter vacuum state with $\omega\sim -1$ and $\dot{H}\sim 0$ occurs in usual four spacetime dimensions. In the present study, for such a vacuum state, we observe an exponential expansion form of the universe in the standard four spacetime dimensions, using D-dimensional equations of the $f(R)$-gravity theory. Moreover, for a possible case, in which the universe can be in the quintessential phase before the matter creation regime (the super inflation scenario permits such a case), we obtain a 10-dimensional structure of the universe with the dominating quadratic term αR^2. Next, we observe that 6 additional dimensions were compactified in the deceleration regime.

Furthermore, it should be emphasized here, that while studying on the usual inflation scenario, it is observed that, after the quasi-de Sitter vacuum state, the expansion was occurring in 4 spacetime dimensions which means the universe directly transits to the deceleration regime. But, the super accelerated phase interval $-1<\omega<\frac{1}{3}$ of the super inflation scenario shows possible creation of the extra dimensions. An interesting case arising from this study is that the results coming from the Starobinsky term αR^2 in the inflation regime and the Einstein term R in the deceleration regime are matching with the condition $-\rho=p_d$. Therefore, the condition providing a conservation of the ordinary matter can appear as a fixed field which produces negative pressure in every point of spacetime tissue of the universe.

The compactification condition, including the case $n>0$ for the relation $b=a^{-n}$, states that additional dimensions are squeezed in every point of the usual 4 spacetime dimensions when comparing with the large-scale structure of deceleration regime. As we remarked above, this case is also produced by the field equation $-\rho=p_d$. Therefore, the condition providing the conservation of the ordinary matter can appear as a fixed field which produces negative pressure in every point of the universe. Hence, the EoS parameter $\omega_{extra}=\frac{p_d}{\rho}$, which can be written for extra dimensions, shows a de Sitter type expansion with $\omega_{extra}=-1$. The interesting value of the EoS parameter mentioned above, which remains constant from the vacuum state of the early universe to the late-time dust epoch, may express the effect of dark energy that causes the expansion of the universe. Hence, after the dust epoch of the universe, according to the observational dataset, we know that the usual spacetime structure did not break down. Furthermore, recent astrophysical observations have shown that the present universe with the de Sitter expansion can be studied with a model including the famous cosmological constant Λ which is expressed with the EoS $\omega_{extra}=-1$ (for instance the ΛCDM model). In other words, it is possible to state that the source of the standard cosmological constant can be extra dimensions which are squeezed in large-scale structure of the universe. Of course, in a separate study, it can be argued that some $f(R)$ gravity models (for example $f(R)=\frac{1}{R}$ type scenario) or other modified gravitational theories can be used to explain whether or not the field relation $-\rho=p_d$ is achieved. If this field equation is protected during all historical expansion regimes of the universe, one can state that the repulsive contribution coming from the extra dimensions due to the negative value of pressure may prevent the universe from collapsing in itself because of the attractive effect of the ordinary matter. From a general perspective, one can conclude that the additional dimensions may be responsible for the speedy expansion phase of the universe after the equilibrium state of the deceleration phase. Nevertheless, in this study, we have shown that the relation $-\rho=p_d$ is protected during the period that exists between the inflation and deceleration regimes by making use of the Starobinsky and Einstein models, respectively.

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