Physics Letters B 834 (2022) 137420

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Progress eras of the universe with spacetime dimensions

A.I. Keskin

Department of Electrical and Electronic Engineering, Faculty of Engineering, Sirnak University, Sirnak, TR-73000, Turkey

A R T I C L E I N F O

Article history:
Received 4 July 2022
Received in revised form 26 August 2022
Accepted 29 August 2022
Available online 31 August 2022
Editor: P. Brax

Keywords:
Extra dimensions
Inflation
Deceleration
String cosmology

A B S T R A C T

In this study, we examine on the expansion history of the universe from the inflationary era to the late-time phantom era in the framework of higher dimensional $F(R)$ gravity. Following the unified first law of thermodynamics method, we find a general entropy expression of the gravity theory for the apparent horizon of the universe. Based on the assumption that the dark energy mystery is closely related to the dimensions of spacetime, we research the progress eras of the universe according to the opening of the space-time dimensionality. In a more general framework, we consider the space time manifold as $n+1$-dimensional spherically symmetric form and discuss the phases of the universe through the validity of entropy. During the inflation, we obtain a 10 dimensional spacetime model which is estimated by the string theory. However, while in the radiation era it is obtained 5 dimensional Kaluza-Klein spacetime, in the dust era 4 spacetime dimensions within the framework of entropy keep standing. At small curvatures of spacetime, a process occurs in which a metric deformation is observed with the negative value of the equation of the state (EoS) parameter, $w < -\frac{1}{3}$. This leads to the release of dark energy from the extra dimensions resulting in a 10 dimensional universe. Since the currently observable universe is in four dimensional spacetime form, this fact indicates that the universe needs to pass into a new phase. Hence, de Sitter phase $w = -1$ of the universe gives 10 dimensional spherically symmetrical spacetime, but here the deformation disappears. Therewithal, we find that 4 dimensional universe model emerges in the phantom region without disturbing the spherical and symmetrical structure of the universe. This means that the late-time accelerated expansion of the universe could be due to dark energy emanating from the extra dimensions.

1. Introduction

The gravitational force, which is an effective phenomenon at the large scales of the universe, is one of the fundamental forces of nature. Einstein's proposal took theoretical cosmology a step further with the idea that the gravitational effect is a curvature of spacetime rather than a force. Nevertheless, observations for the last 20 years have concretely carried theoretical cosmology to the desired level. The going away of galaxy clusters, measurements of light-curves arising from supernova explosions and observations of the cosmic microwave background (CMB) have made it almost certain that the universe is accelerating [1–9]. In spite of the attractive effect of gravity, this acceleration is tried to be explained by a fluid which is named "dark energy" has a repulsive effect, due to negative pressure properties. This energy is described by the boundary condition $w < -\frac{1}{3}$ of the equation of state (EoS) parameter, the quintessence field, which also corresponds to a dynamical form of dark energy and has a theoretical configuration coming from observations [9]. Also de Sitter vacua, $w = -1$ or ΛCDM model, is another type of dark energy which is identified by a positive cosmological constant $\Lambda > 0$ and it has been considered that the present universe is accelerating with $\Lambda > 0$. According to the string theory, the universe is under the effect of the quintessence dark energy, and dark energy solutions in the theory do not give positive value of Λ but produce negative Λ solutions corresponding to anti-de Sitter vacua [10,11]. In this context, the compatibility of low-energy effective field theories with quantum gravity should meet some criterions recently discussed in the swampland conjecture. In low-energy effective field theories de Sitter solutions together with positive cosmological constant are important for the swampland conjecture [12–14] in way of consistency of any quantum gravity theories. If the string theory is the ultimate quantum gravity theory, it is needed its consistency with the swampland conjecture to assort with string landscape. In this study, inspired by the dimensionality of the string theory, in the framework of the higher-dimensional

E-mail addresses: a.ihsankeskin@sirnak.edu.tr, alikeskin039@gmail.com.

https://doi.org/10.1016/j.physletb.2022.137420

string $F(R)$ gravity, dimensionality evolution of the universe is investigated through one phase to another one, especially from the deceleration phase to the acceleration phase besides that the inflationary era [15–20]. It is obtained de Sitter vacua with positive cosmological constant and observed a symmetry between the early and the late-time universe.

On the other hand, dark energy can be also explained by using modifications of gravity theories, i.e. $f(G)$, $F(T)$ and $F(R,G)$ theories as well as the scalar tensor theories and Brane world scenario etc. [20–41]. It is observed that these modifications can describe the theoretical framework of dark energy in the background of the cosmological observations in a cogent and accurate way, especially with spatially flat Friedmann-Robertson-Walker (FWR) universe model. In particular, the inflationary [42] cosmological setup [43] with an exponential expansion, which is known as the quasi-de Sitter expansion, seems to be a successful scenario. Comparing with Einstein's R curvature scalar, the R^2 modification term [43], which prevails at the high-energy scale of spacetime, sharply describes the behavior of the early inflation in agreement with observations. It should be noted that here, in this study we use the higher dimensional R^2-term to show the inflationary era. Hence, in the presence of extra dimensions, the analysis of the early inflation [45] with this term, and studies on the late-time dark energy of the universe can be seen in the literature [44,46,47]. In the study [45], the authors obtained a 10 dimensional universe model and considering the super inflation scenario for the early universe they showed that 6 spatial dimensions compactified on four dimensions in the deceleration eras (radiation and dust eras) so the expansion is shaped by the known four dimensional spacetime. During the formation of galaxies, the universe gains heavy mass (due to attractive effects) in the dust era which may cause the universe is collapsing on itself, so surviving of the present universe may be due to effects of dark energy compactified on the extra dimensions. Here, relevant to the crossing of the universe from the dust era to the quintessence field leading to an accelerated expansion, we can state that the extra dimensions curled on four spacetime dimensions can open again and release dark energy. The present study put forward this case with a small deformation of the metric function in the background of cosmological entropy. This may be an answer to the question of why the late-time universe is accelerating. In this sense, we focus on gravitational entropy law that is associated with the thermodynamics of a black hole. It is known that a great step in this direction was considered by Jacobson [48] who showed from a thermodynamic point of view that the equation of state of spacetime is exactly the standard Einstein equations. In this direction, a lot of studies have been carried out showing the relationship between thermodynamics and gravity [49–82]. In particular, Hayward [59–61] worked on the thermodynamics of a dynamic black hole, where he defined the outertrapping horizon for the black hole and put forward the unified first law to describe the apparent horizon of the FLRW universe. In his approach, the Clausius relation of a dynamical black hole can be written as $< A\Psi, \xi >= \frac{k}{8\pi G} < dA, \xi >$, with the energy supply vector ψ defined in the unified first law method. The Clausius relation of the dynamic black hole is given in the form $Q = TdS$, where Q and T respectively show heat flow and the Unruh temperature measured by an accelerated observer just inside the horizon. By using the entropy formula similar to a black hole horizon for the apparent horizon of the FLWR universe, the authors [62–66] showed that the Clausius relation is valid. However, the deriving universal entropy for different gravity theory can also be found in the studies [67,68].

In the present study, using the Clausius relation we derive an entropy expression for the apparent horizon of the spherically symmetrical FRW universe model within a thermodynamics equilibrium. Moreover, we determine the entropy of matter inside the horizon to obtain a generalized entropy inequality. In the framework of the entropy inequality, we research for the validity of the theoretical findings produced by the field equations of the gravity for the known eras of the universe. It is observed that the early-time quasi-de Sitter vacua and the quintessence dark energy is potently connected with the late-time exact de Sitter vacua and the quintessence dark energy, although it is used different $F(R)$ models. This is a symmetry for dark regimes of the universe. It is discussed this symmetry of the mechanism coming from the entropic analysis in view of the swampland proposed to determine the bounds of the string landscape [12–14] (of any effective quantum gravity) which can be described real universe. It is concluded that the quintessence type of dark energy [11,83,84], which naturally arises from the string-$F(R)$ solutions, plays a critic role on evolution of the universe. This is naturally emerges in the string theory, so we can say that the entropic approach may be another candidate heading for drawing the border of the landscape in the swampland. Also we discuss observables to be related to the presented mechanism and it is identified the deceleration-acceleration redshift transition, consequently we set this transition occurs at the redshift $z_t = 0.67$.

2. Higher dimensional $F(R)$ Friedmann equations

In this section, we start with the action of $F(R)$ gravity written for $D = 4 + d$ dimensions [45,85],

$$S = \frac{1}{2k_D^2}\int d^D x\sqrt{-g_D}F(R) + L_m^D \tag{1}$$

where $\kappa_D^2 = 8\pi G_D$, G_D is D-dimensional Newton constant, L_m^D shows a matter action, $F(R)$ is an arbitrary function of the Ricci scalar constructed from D-dimensional metric g_{AB} $(A, B = 0, ..., d+3)$ and d is the number of the extra dimensions. From the action (1) the following field equations are obtained

$$R_{AB}F(R) - \frac{1}{2}F(R)g_{AB} - (\nabla_A\nabla_B - g_{AB}\nabla^2)F_R = k^2 T_{AB}, \tag{2}$$

where $\nabla^2 = \nabla_A\nabla_B$ is D'alembertian operator in D-dimensional framework. The energy- momentum tensor, T_{AB}, which describes a perfect fluid flowing inside the Universe, is given by [46,86]

$$T_B^A = diag[-\rho(t), p(t), p(t), p(t), p_d(t), ...p_d(t)], \tag{3}$$

where $p_d(t)$ is the pressure along the extra dimensions. We consider a higher dimensional FLRW metric [86]

$$ds^2 = -dt^2 + a^2(t)[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)] + b^2(t)\gamma_{ab}dy^a dy^b \tag{4}$$

where γ_{ab} (with $a, b, ... = 4, 5, ...d$) and $b(t)$ show the metric and the scale factor of the extra dimensions, respectively. The Riemann tensor for the extra dimensions is chosen the maximally symmetric [86] as $R_{abcd} = k(\gamma_{ac}\gamma_{bd} - \gamma_{ad}\gamma_{bc})$, where the choosing $k = 0$ yields a manifold for the extra dimensions. However, in further calculations, we consider a manifold that visible universe dimensions are assumed to be a flat case, i.e., $K = 0$, according to the recent observational data [3–9]. According to the metric (4), higher dimensional Friedmann equations are obtained as the following [45]

$$-\frac{3\ddot{a}}{a}F_R + \frac{1}{2}F(R) + \frac{3\dot{a}}{a}\dot{F}_R - \frac{d\ddot{b}}{b}F_R + \frac{d\dot{b}}{b}\dot{F}_R = \kappa_D^2\rho \tag{5}$$

$$(\frac{\ddot{a}}{a} + \frac{2\dot{a}^2}{a^2})F_R - \frac{1}{2}F(R) - \frac{2\dot{a}}{a}\dot{F}_R - \ddot{F}_R + \frac{d\dot{a}\dot{b}}{ab}F_R - \frac{d\dot{b}}{b}\dot{F}_R = \kappa_D^2 p \tag{6}$$

$$(\frac{\ddot{b}}{b} + \frac{(d-1)\dot{b}^2}{b^2} + \frac{3\dot{a}\dot{b}}{ab})F_R - \frac{1}{2}F(R) + (1-d)\frac{\dot{b}}{b}\dot{F}_R - \ddot{F}_R - \frac{3\dot{a}}{a}\dot{F}_R = \kappa_D^2 p_d \tag{7}$$

where $F_R = \frac{dF(R)}{dR}$ with higher dimensional Ricci curvature scalar,

$$R \equiv R + R^d = 6\dot{H} + 12H^2 + \frac{2d\ddot{b}}{b} + \frac{6d\dot{a}\dot{b}}{ab} + d(d-1)\frac{\dot{b}^2}{b^2}. \tag{8}$$

Furthermore, the conservation law ($\nabla_A T^A_B = 0$) satisfies that [86]

$$\dot{\rho} + 3H(\rho + p) = -d\frac{\dot{b}}{b}(\rho + p_d). \tag{9}$$

On the other hand, we can write the equations (5), (6) and (7) in the form of the Einstein's field equations [45]

$$H^2 = \frac{\kappa_D^2}{3}\rho_t, \tag{10}$$

$$\dot{H} = -\frac{\kappa_D^2}{2}(\rho_t + p_t), \tag{11}$$

which show the continuity equation,

$$\dot{\rho}_t + 3H(\rho_t + p_t) = 0. \tag{12}$$

Here, we define the quantities $\rho_t = [\rho_m + \rho_{eff}]$ and $p_t = [p_m + p_{eff}]$, with the effective energy density and the pressure

$$\rho_{eff} = \frac{1}{\kappa_D^2 F_R}(3\dot{H}F_R + 6H^2F_R - \frac{1}{2}F(R) - 3H\dot{F}_R + \frac{d\ddot{b}}{b}F_R - \frac{d\dot{b}}{b}\dot{F}_R, \tag{13}$$

$$p_{eff} = \frac{1}{F_R}[-p_d + \frac{1}{\kappa_D^2}(\{\frac{\ddot{b}}{b} + \frac{(d-1)\dot{b}^2}{b^2} + (3-d)H\frac{\dot{b}}{b} - 3\dot{H} - 6H^2\}F_R + (\frac{\dot{b}}{b} - H)\dot{F}_R)]. \tag{14}$$

Here, $H = \frac{\dot{a}}{a}$ shows the Hubble parameter of the observed universe. Also, the matter quantities are given by $\rho_m = \frac{\rho}{F_R}$ and $p_m = \frac{p}{F_R}$. The sum of the two equations is

$$\rho_{eff} + p_{eff} = \frac{1}{\kappa_D^2 F_R}\{\ddot{F}_R - \frac{\dot{a}}{a}\dot{F}_R + d(\frac{\ddot{b}}{b} - \frac{\dot{a}\dot{b}}{ab})F_R\}. \tag{15}$$

We will use this summation when deriving the entropy of the apparent horizon.

3. The entropy of horizon

In this section, we shall derive the entropy of the apparent horizon for the FRW universe model. For this, we separate the metric (4) into two parts, i.e., the spatial and time parts. In refs. [62–66], the authors considered a general metric which is a generalized definition of four dimensional black hole on the trapping horizon, namely $(n+1)$-dimensional spherically symmetric FRW metric (our notation is $D - 1 \equiv n$),

$$ds^2 = h_{ab}dx^a dx^b + \tilde{R}^2 d\Omega^2_{n-1}, \tag{16}$$

where the metric of $(n-1)$-dimensional sphere is $d\Omega^2_{n-1}$. Also, $\tilde{R} = ar$ is the radius of the sphere and $h_{ab} = diag(-1, a^2(t)\frac{dr^2}{1-Kr^2})$ is the two dimensional metric, which includes one time ($x^0 = t$) and one space ($x^1 = r$) dimensions of the visible universe. Hence, the apparent horizon ($\tilde{R}$) can be determined over (t, r) coordinates [50–55]. The solution of the $\partial_+\tilde{R}|_{\tilde{R}=R_T} = 0$ yields the horizon formula, which coincides with the trapping horizon R_T [64,66,87], as follows:

$$\tilde{R} = R_T = (H^2 + \frac{k}{a^2})^{\frac{-1}{2}}. \tag{17}$$

Here, $\partial_+ = -\sqrt{2}(\partial_t - \frac{\sqrt{1-Kr^2}}{a}\partial_r)$ is future pointing null vector. In a form similar to the metric given by (16) the metric (4) at hand can be written as follows:

$$ds^2 = h_{ab}dx^a dx^b + \tilde{R}^2(d\theta^2 + \sin^2\theta d\phi^2)] + b^2(t)\gamma_{ab}dy^a dy^b. \tag{18}$$

Since the radius of the horizon is determined by (t, r) coordinates, the selection of b is an important factor to arrive at the metric which describes the spherically symmetric spacetime given by (16). In this study, we assume the scale factor relationship $b = a^{-n}$ [86] that provides a connection between the two parts of spacetime, namely extra and visible dimensions. With this argument, it can be observed that the spherically symmetric structure of spacetime is preserved with some values of n, otherwise it is mentioned a deformation. For instance, together with the fixed value $n = -1$, the metric (18) and the metric (16) have the same form, so D-dimensional spherically symmetric spacetime emerges. In the case of $n > 0$, we come across a four-dimensional spherically symmetric spacetime form. But, all the values in the range $n < 0$ apart from the constant value $n = -1$ disrupt the homogeneous and symmetrical structure of the above metric and cause a deformation. For the $n = -1$ case, it should be noted that if the number of dimensions is $d = 1$, then the five-dimensional Kaluza-Klein metric, which is a special form of the D-dimensional metric, is obtained [45].

Let us now turn to the thermodynamic framework to expand our discussion. The unified first law (UFL) of thermodynamics in the Clausius form $\delta Q = TdS$ can be written as [62–66]

$$\delta Q = F_R < A\psi_m, \xi >= \frac{F_R \kappa}{8\pi G_D} < dA, \xi > - F_R < A\psi_{eff}, \xi >= TdS, \tag{19}$$

where in our case the work density $W = -\frac{1}{2}T^{ab}h_{ab}$ and the energy supply vector $\psi = T_a^b \partial_b \tilde{R} + W\partial_a \tilde{R}$ are given by

$$\begin{aligned} W &= W_m + W_{eff} = \frac{1}{2F_R}(\rho - p) + \frac{1}{2F_R}(\rho_{eff} + p_{eff}), \\ \psi &= \psi_m + \psi_{eff} = -\frac{1}{2F_R}(\rho + p + \rho_{eff} + p_{eff})H\tilde{R}dt + \frac{1}{2F_R}(\rho + p + \rho_{eff} + p_{eff})adr. \end{aligned} \tag{20}$$

The $(D-2)$-Dimensional surface area, A, and $(D-1)$-Dimensional volume, V, are respectively given by [62–64]

$$A = B\tilde{R}^{D-2}, \quad V = \frac{B}{D-1}\tilde{R}^{D-1}, \tag{21}$$

where $B = \frac{2\pi^{\frac{D-1}{2}}}{\Gamma(\frac{D-1}{2})}$ with the Gamma function $\Gamma(x) = (x-1)!$. In (t, r) coordinates, any tangent vector to the apparent horizon is given by [62–65]

$$\xi = \frac{\partial}{\partial t} - (1 - 2\epsilon)Hr\frac{\partial}{\partial r} \tag{22}$$

Furthermore, the surface gravity and the Hawking temperature on the apparent horizon are defined by

$$\kappa = -\frac{(1-\epsilon)}{\tilde{R}}, \qquad T = \frac{|\kappa|}{2\pi}, \tag{23}$$

where we assume negative surface gravity with $\epsilon = \frac{\dot{\tilde{R}}}{2H\tilde{R}} < 0$, which coincides with inner trapping horizon. Using surface area in (21) the resulting inner product terms given by (19) is as the following:

$$\delta Q = F_R < A\psi_m, \xi >= \frac{2\epsilon(D-2)\kappa HB\tilde{R}^{D-2}F_R}{8\pi G_D} - AH\tilde{R}(\rho_{eff} + p_{eff})(1-\epsilon). \tag{24}$$

Extracting the temperature and again writing in inner product form, we obtain the following equation,

$$\delta Q = T < \frac{F_R dA}{4G_D} - 2\pi BH\tilde{R}^D(\rho_{eff} + p_{eff})dt, \xi >= TdS_A, \tag{25}$$

and from which the time derivative of the entropy is obtained as follows,

$$\dot{S}_A = \frac{F_R B(D-2)\tilde{R}^{D-3}\dot{\tilde{R}}}{4G_D} - 2\pi BH\tilde{R}^D(\rho_{eff} + p_{eff}), \tag{26}$$

where $(\rho_{eff} + p_{eff})$ is given by (15). Here, for some limiting cases, the equation transforms into known some forms of entropy. For instance, when inserting $D = 4$, $G_D = G$ and the four-Dimensional Einstein term $F(R) = R$ into (26), then by integration, the Bekenstein entropy relation $S_A = \frac{\pi\tilde{R}^2}{G}$ is restored. For only $F(R)$ gravity with $D = 4$, we obtain the Wald entropy relation $S_A = \frac{\pi\tilde{R}^2F_R}{G}$ [88] without the second term $(\rho_{eff} + p_{eff})$, so the second term is a modification term that includes the additional dimensions part.

3.1. Generalized second law of thermodynamics

In order to discuss validity of the cosmological entropy law in the eras of the universe, we must consider the matter content flowing inside the horizon. When the cosmic spacetime as a whole is considered as a thermodynamics system, its constituent sub-discrete systems are the matter contents moving in its dimensions. Therefore, for an universal cosmological entropy law setup, the entropy of the matter components on the horizon should be added to the entropy of the apparent horizon and so such a system should be considered. Hence, the matter entropy change can be found by using thermodynamics Gibb's equation,

$$T_m dS_m = d(\rho V) + pdV. \tag{27}$$

Here, the energy density and the D-dimensional volume are given by (9) and (21), respectively. Using these quantities we find the time derivative of the matter entropy as follows,

$$\dot{S}_m = \frac{4\pi B\tilde{R}^{D+1}\rho H}{(D-1)(2\pi\tilde{R}-\dot{\tilde{R}})}[\{-3H(1+w)+nd(1+w_{ex.})\}H+\dot{\tilde{R}}\tilde{R}^{-1}(1+w)(D-1)] \tag{28}$$

where we define the EoS parameters of the matter and the additional dimensions, respectively, $w=\frac{p}{\rho}$ and $w_{ex.}=\frac{p_d}{\rho}$ [45] Also, we assumed a thermodynamic equilibrium between the matter and the horizon, i.e., $T_m=T=\frac{\kappa}{2\pi}$. In fact, it is important to establish a thermodynamic equilibrium so that no deformation is observed in the space-time geometry [87]. As a result, the generalized entropy law ($S_t=\dot{S}_A+\dot{S}_m$) of the universe can be written as follows:

$$\frac{F_R B(D-2)\tilde{R}^{D-3}\dot{\tilde{R}}}{4G_D}+2\pi B\tilde{R}^D\{-(\rho_{eff}+p_{eff})H+\frac{2\tilde{R}H\rho}{(D-1)(2\pi\tilde{R}-\dot{\tilde{R}})}[Hdn(1+w_{ex.}) \\ -3H(1+w)+(D-1)\tilde{R}^{-1}\dot{\tilde{R}}(1+w)]\}\geq 0. \tag{29}$$

4. Progress eras of the universe from the entropy perspective

In order to discuss all the evolution of the universe from the beginning inflation to the late-time dark energy eras in a unified form [20], we take the higher dimensional model $F(R)=\alpha R^2+R+\beta R^{-m}$, with $m=1$ and positive coupling constants α, β. At higher curvature regime (early inflation) the higher dimensional Starobinsky term αR^2 becomes dominated and at low curvature case (dark energy) the higher dimensional term $\frac{1}{R}$ [89] becomes dominated, when comparing with the higher dimensional Einstein term R. Hence, this model can describe the historical evolution of the universe, which can be broadly divided into three stages. In particular, It is worth emphasizing that the model $R+\alpha R^2$ is invariant [94,95] under string duality transformation when existence of Noether Symmetries [96]. In this regard, it is worth stressing that in the entropic analysis the $1/R$ model in the late-time universe gets a reflection symmetry or exhibits same dynamics with the term αR^2 describing the early-time universe. Hence, the entropic approach may come into an important role to reveal viable $F(R)$ model coming from the string landscape.

4.1. The inflation of the universe

In this title, we deal with the super inflation scenario [90–92], which assumes that the inflation realizes in the three phases. The first phase is a quasi-de Sitter vacuum state. The second phase is a quintessential case, where the universe expands with the power-law solutions of the scale factor $a=t^{h\equiv\frac{4}{3(1+w)}}$. The latter one is a phase that the normal matter is created. The choice of this inflation scenario is because it predicts the existence of the additional dimensions for the beginning of the universe [45] and includes the quintessence dark energy appearing in the string theory.

4.1.1. A quasi-de Sitter vacuum state

For the quasi-de Sitter vacuum state, we have the conditions $\dot{H}_0\sim 0$ and $w\sim -1$. Using the scale factor relationship $b=a^{-n}$ [86] into (8), the curvature scalar is obtained as follows:

$$R=(12+dn^2-6dn+d^2n^2)H_0^2=JH_0^2. \tag{30}$$

Inserting the term αR^2 into (29), we find

$$8\pi H_0^{-D+1}\{\frac{\rho H_0 dn(1+w_{ex.})}{(d+3)}-\frac{\pi ndH_0^2(n+1)}{\kappa^2}\}\geq 0. \tag{31}$$

When the EoS parameter of additional dimensions is at the point $w_{ex.}=-1$ or $p_d=-\rho$, we obtain $n=-1$ [45]. That is, $w_{ex.}=-1$ and corresponding to the fixed point $n=-1$, are the fixed field of the scenario [45], with $\alpha>0$ [20] (so that anti-gravity does not appear). Herein, the EoS parameter $w_{ex.}=-1$ of the additional dimensions provides both the conservation of the matter appearing in (9) and symmetrical structure of the metric (18). Hence, we can write the entropy inequality given by (29), as follows:

$$\dot{S}_t=-\frac{8\pi^2 ndH_0^{-D+3}}{\kappa^2}(n+1)\geq 0. \tag{32}$$

As seen that, even if it is mentioned existence of the additional dimensions, it is observed that the change of the entropy with cosmic time is zero. Since observed scale factor increases exponentially $a=a_0[e\times p(\frac{16\pi G_D\rho_0}{\alpha d(d+4)(d+3)^2})^{\frac{1}{4}}t]$ from eq. (5), we remark the two possibilities here. The first is an information read at $d=0$ which emerges, independently of cosmic time t, a singularity on scale factor $a\to\infty$ with zero entropy. This means that the initial universe was not in the known four dimensional form. The second one points to a possible $d+4$ dimensional symmetrical multiples spacetime. Here, we can say due to zero entropy that the spacetime was originally in a regular and symmetrical structure since the normal matter, which moves in spacetime dimensions and probably causes disorder, was not yet created. According to our theoretical model, it is possible to state that the vacuum energy propagated in spatial d dimensions may correspond to dark energy mystery. However, due to zero entropy this form of dark energy is not dynamical. It should be noted that if $d=0$, non-dynamical form can emergence because of uncertainty on the scale factor. This is a fluid that may not be detected from observations, and if so, it is needed another type of dark energy emerging the present universe dynamics. This may be the quintessence type of dark energy which has the dynamical properties and naturally reside in the string landscape [93].

4.1.2. The quintessence expansion phase

The curvature scalar has the form

$$R = t^{-2}(2dnh + 12h^2 - 6h + dn^2h^2 + d^2n^2h^2 - 6dnh^2) = Lt^{-2}, \tag{33}$$

with the power-law scale factor $a = t^{h \equiv \frac{4}{3(1+w)}}$. Inserting the inflation term, αR^2, into (29) with fixed value $n = -1$ (where the quintessence phase is also a time-varying vacuum state) which provides the conservation of the matter with $p_d = -\rho$ in eq. (9), we obtain

$$\frac{\alpha L(D-2)h}{G_D} + \frac{8\pi\rho_0[hdn(1+w_{ex.}) + (1+w)(-3h+D-1)}{(D-1)(2\pi t - 1)} \geq \frac{4\pi M t^2}{\kappa_D^2}, \tag{34}$$

where $M = 6 + 2h + dnh + dn^2h^2 + dnh^2$, and ρ_0 is a integral constant coming from the scale factor power-law solution of eq. (9) with fixed field $p_d = -\rho$. Also from eq. (5) we obtain

$$\alpha L = \frac{16\pi\rho_0 G_D}{h[d^2h + 3hd - 6d - 18]}, \tag{35}$$

and inserting it into eq. (34) we find

$$\frac{2(d+2)}{[d^2h + 3hd - 6d - 18]} + \frac{(1+w)[3(1-h)+d]}{(d+3)(2\pi t - 1)} \geq \frac{(6 + 2h + dnh + dn^2h^2 + dnh^2)t^2}{16\pi\rho_0 G_D}. \tag{36}$$

It is observed in very small value of cosmic time t $(0 < t \ll 1)$ that the equality above is valid for $d > 0$ or $d = 6$, when $8\pi\rho_0 G_D \sim 1$ or $8\pi\rho_0 G_D > 1$ with the range $h > 2$. Especially the range $h > 2$ corresponds to the $w < -\frac{1}{3}$ interval showing the quintessence phase of the scenario. However, the validity of the inequality for a precise control depends on the denominator of the first term which not being zero, due to the positive definition of matter density appeared in eq. (35) (to avoid anti-gravity see Ref. [20]). This condition leads to $h > \frac{6}{d}$, and with $-1 < w < \frac{1}{3}$ we obtain the range $0 < d \leq 6$ [45], which is a prediction of the string theory. After the radiation-like particles are created, where $w \sim \frac{1}{3}(h \sim 1)$, the inflation ends and the universe enters the radiation era with the Einstein term R appeared in the model. The quintessence is a field (in our study, the quintessence field is described by the range $-\frac{1}{3} < w < -1$ rather than a scalar field) that is originally produced by the homogeneous scalar field (time dependent), and it depicts dark energy in a dynamic framework when comparing with the cosmological constant arising from the vacuum state. Hence, it differs from the vacuum state due to its time-varying nature and inherently contains the degrees of freedom of the string theory [93]. Considering that a 10 dimensional universe model is deduced in the quintessence field and its compatibility with the string theory in this point, it becomes clear that the inflation era should cross through, at least, one phase after the vacuum state in order to understand dark energy mystery. The effect on spacetime due to the dynamic nature of the quintessence field is that it opens up the extra dimensions compactly buried at every point in spacetime or causes the creation of the additional 6 dimensions, perhaps allowing the universe to become a ready made for the creation of matter particles. As well as the number of opened dimensions is 6 according to the mechanism, a 10-dimensional non-degenerate, symmetrical and homogeneous universe model is obtained and depicted, with the metric function (18). Although this case approximately indicates the range $2 < h$ in the entropy inequality, the value $h = 1$ violates the inequality, and this case notifies that the universe should be in a new phase state and conjecture.

4.2. The deceleration era of the universe

The curvature scalar is in the form of (33) with the scale factor solutions $a = t^h$. Using eqs. (5), (15) into eq. (29) we obtain

$$16\pi^2\rho_0 h^{-D} t^{D-3}\{\frac{[(d+2) - dn(1+nh+h)]}{h(6 - 6dn + d^2n^2 - dn^2)} + \frac{2(d+3(1-h))}{3(d+3)(2\pi t - 1)}\} \geq 0, \tag{37}$$

where $h = \frac{2}{3(1+w)}$. On the other hand, the condition $p_d = -\rho$ gives $n = \frac{3(1-h)}{5+3h}$, with $d = 6$. It is clear that the radiation phase $h = \frac{1}{2}$ and the dust phase $h = \frac{2}{3}$ show the range $n > 0$. On the other side, in view of the entropy it is concluded that the early radiation phase $(2\pi t < 1)$ is valid up to $d = 5$ dimensions. Integer numbers in $d > 5$ interval violate the entropy inequality, so five spacetime dimensions emerge in the radiation era. This is a Kaluza-Klein proposal which attempts to unify 4 dimensional Einstein gravity and Maxwell's theory of electromagnetism within a five-dimensional framework [97–99]. Since the universe is in an era filled with relative particle density and electromagnetic radiation, electromagnetic forces are inseparable from gravity and in a unified form due to the existence of the fifth dimension. The metric (18) refers approximately to the form of the Kaluza-Klein metric because the other additional 5 dimensions are in compact form.

On the other side, for the late-time dust phase $h = \frac{2}{3}$ it is observed that the compactification condition of the additional dimensions realizes with the values $d = 6$ from entropic constraint (37). This also means that the electromagnetism is separated from gravity, and dark energy is stored in the extra dimensions due to $w_{ex.} = -1$. One of the main points to be noted is that the structure of the universe is similar to its present form in this phase.

4.3. The additional dimensions release dark energy into the late-time quintessence era with a deformation of the metric function

In this section, we deal with $\sim \frac{1}{R}$ term which is dominate at small curvatures of spacetime. The curvature scalar is in the form of (33) with the solutions $a = t^{h \equiv \frac{2}{3(1+w)}}$. The matter should be conserved in this phase of the universe as well, so the EoS parameter of additional dimensions $p_d = -\rho$, with the term $\sim \frac{1}{R}$, produces the equation,

$$\beta L^{-2}t^2[-12+3h-3h^2-3nh-dnh-3nh^2]=0. \tag{38}$$

We find the solution,

$$n=-\frac{h^2-h+4}{h(1+\frac{d}{3}+h)}, \tag{39}$$

where $d=6$. For acceleration condition $h>1$ or $w<-\frac{1}{3}$, we obtain the range

$$-1<n<0, \tag{40}$$

which leads to an instability on the metric (18). This means that dark energy stored in the deceleration era releases into the visible spacetime dimensions during the late-time quintessence era. From eq. (29) the entropy can be written as the follows,

$$8\pi^2 h^{-D+1}t^{D-3}\{\frac{-h\beta L^{-2}t^4(D-2)}{8\pi G_D}-\frac{M}{8\pi G_D}+\frac{2\rho_0(1+w)(d+3(1-h)}{(d+3)(2\pi t-1)}\}\geq 0, \tag{41}$$

where $M=12-4h+dhn+dn^2h^2+dnh^2$. Using eq. (5) in which we obtain $\kappa_D^2\rho_0=\beta L^{-2}Ct^4$, the inequality above can be rewritten as follows,

$$2\pi\rho_0 h^{-D+1}t^{D-3}B[\frac{-(d+2)h}{C}-\frac{M}{8\pi\rho_0 G_D}+\frac{4(-3h+d+3)}{3h(d+3)(2\pi t-1)}\}\geq 0, \tag{42}$$

where $C=-18h+9h^2+6dnh+\frac{3dn^2h^2}{2}+\frac{d^2n^2h^2}{2}-3dnh^2$. The matter energy density must be positive that requires the condition $C>0$. All values in $h>1$ interval provides the range $C>0$, together with the n value given by (39). The simpler form of the entropy can be written as follows:

$$\frac{-8h^2}{C}+\frac{4(3-h)}{9(2\pi t-1)}\geq\frac{Mh}{8\pi\rho_0 G_D}. \tag{43}$$

Taking the present value of the Hubble parameter $H_0=1.37187\times 10^{-33}$ eV [9] into account and using $8\pi\rho_0 G_D\sim 3\Omega_0 H_0^2$, we find the entropy (43) is valid about the range $h>1.19$. Here Ω_0 is matter density parameter which is $\Omega_0\simeq 0.3$ [9].
While the value $n=-1$ in the metric (18) describes a $d+4$ dimensional universe model, the case arising from the $n>0$ interval show that the extra dimensions are compactified on the visible four spacetime dimensions. Although the difference is only in their dimension numbers, both values describe the geometry of a symmetric spacetime. For the case in here, it is clearly required $n>-1$ for the acceleration. On the other hand, all the values in the range of $n<0$ except $n=-1$ mean existence of a deformation on metric. We can read this deformation from the entropy expression which is obtained on the basis of the symmetrical spacetime. In other words, the range given by (40) provides the validity of entropy with the accelerated expansion condition, $h>1$, but the entropy gets a constraint in $h>1.19$ interval, and for $h=1$ the inequality loses its validity.
The late-time quintessence field plays a critical role related to the survival of the universe in which hidden additional dimensions are reopened from visible four spacetime dimensions and create a repulsive effect on spacetime before a collapsing case that can be caused by attractive gravitational effects arising from the dust phase. Therefore, the matter, whose density decreases in the deceleration eras as the universe expands, becomes recessive when comparing with quintessence dark energy, and as a result, the universe accelerates with the dominance of the quintessence dark energy.
it is worth stressing that for both the early inflation and the late-time acceleration the dynamical structure of the quintessence field plays the role providing opening of the additional dimensions. It is also necessary to emphasize that, according to the entropy constraint, the $\alpha R^2+\beta R^{-1}$ model of the gravity can be located in the string landscape because they exhibit the same dynamics. Due to this symmetrical reflection, the entropic analysis may be an effective approach which draws a border for vast the string landscape in the swampland.

4.4. de Sitter point and phantom era

The de sitter point $w=-1$ leads to an exponential expansion of the scale factor of the visible universe. Using the constant Hubble solution H_0 and exact de Sitter point of the extra dimensions $w_{ex.}=-1$ or $p_d=-\rho$ we obtain the equation,

$$3\beta J^{-2}H_0^{-2}(n+1)=0, \tag{44}$$

where we used (30) and neglected higher derivative of the Hubble parameter. The solution is the point $n=-1$. In this case, an exact linear form of the metric (4) is observed, where $b=a$, which means that we have a 10 dimensional spherically symmetrical spacetime model. Moreover, the same result can be seen by taking the limiting case $w\to -1$ in eq. (39), where n goes to -1. The entropy (29) for this point is as follows:

$$2\pi BH_0^{-D+3}\{\frac{-dn(n+1)}{8\pi G_D}\}\geq 0. \tag{45}$$

This result is the same with the result obtained for early quasi-de Sitter vacua. The approximate value, $n\sim -1$, indicates that the total entropy change of the universe is zero, which means that the Sitter vacua is not a dynamical structure. Also, this indicates a perfect structure where the development of the universe is perhaps completed, because at this point the universe has zero entropy or a constant entropy. It is understood that the universe is completely dominated by dark energy at this point, and thus, it enters inflation-like accelerated expansion again, regaining a spherically symmetrical structure at this point.

Now let's discuss the observables related to this phase. Eq. (5) can be written as $\beta = 450\Omega_\Lambda H_0^4$, where we have introduced density parameter $\Omega_\Lambda = \frac{8\pi G_D \rho_0}{3H_0^2}$. Using the present value of the Hubble parameter $H_0 \simeq 1.37 \times 10^{-33}$ eV and $\Omega_\Lambda \simeq 0.681369$ [9], we find that $\beta \simeq 1080.04 \times 10^{-132}$ eV^4. Also again using the equation $\beta = 450\Omega_\Lambda H_0^4$ we obtain exponential scale factor solutions, $a = a_0[e \times p(\frac{\Lambda}{3})^{\frac{1}{2}}t]$, with a positive definition of the cosmological constant, where we have introduced the cosmological constant as $\Lambda = \frac{\beta}{400\pi G_D \rho_0}$, and from which we obtain $\Lambda \simeq 5,61 \times 10^{-66}$ eV^2. This result agrees with the observation data given by $\Lambda = (4.24 \pm 0.11) \times 10^{-66}$ eV^2 [9].

On the other side, for a phantom cosmological solution, $a = a_0(t_s - t)^{-h}$, with $h > 0$, $a_0 = 1$, we can shift some quantities from eqs. (33) and (38) as $h \to -h$, $t \to (t_s - t)$, so we obtain

$$R = (-2dnh + dn^2h^2 + d^2n^2h^2 - 6dnh^2 + 12h^2 + 6h)(t_s - t)^{-2} = L_*(t_s - t)^{-2}, \quad (46)$$

$$\beta L_*^{-2}(t_s - t)^2[12 + 3h + 3h^2 - 3nh - dnh + 3nh^2] = 0. \quad (47)$$

Here, t_s is the Big-Rip time. From (47) we have the solution

$$n = -\frac{h^2 + h + 4}{h(h-3)}. \quad (48)$$

For visible spacetime dimensions condition $n > 0$, we obtain the $0 < h < 3$ interval. Furthermore, since $h = -\frac{2}{3(1+w)}$, we have the range $w < -1.22$. However, inserting (48) into (43), with the alteration $h \to -h$, the entropy law for the phantom phase is obtained as

$$\frac{-8h^2}{C_*} - \frac{1}{36} - \frac{1}{12h} \geq \frac{M_* h}{64\pi \rho_0 G_D}. \quad (49)$$

This inequality gives the two ranges, i.e., $1 < h < 3$ and $h > 3$ corresponding to $-1.66 < w < -1.22$ and $-1.22 < w < -1$, respectively. But, we deal with the first range $-1.66 < w < -1.22$ due to the constraint of eq. (48). We note that this range will be discussed in detail in the next title.

It is concluded that the 10-dimensional symmetrical structure turns into a 4-dimensional symmetrical structure in the phantom era, where the 6-spatial dimensions are compactified on four spacetime manifold. Hence, the universe should be passing over the point $w = -1$ with the increasing of entropy, according to the present mechanism. The emergence of 4-dimensional form of the universe in a phantom phase suggests that the mechanism has some symmetries. For example, the phantom phase resembles the structure of the universe in the dust phase (we can even say the same), but there is an acceleration state in the phantom phase.

4.4.1. Deceleration-acceleration redshift transition and cosmographic parameters

The modern cosmological observations show that the expansion of the universe switches from decelerating phase to recent accelerating one. In this title, we will discuss the possible observables to be related to the mechanism, transition redshift, which identify cosmological deceleration- acceleration transition. For $F(R)$ gravity the deceleration-acceleration transition redshift was reported in Ref. [100]. The deceleration parameter in terms of the Hubble parameter (H) is given by

$$q = -\frac{\ddot{a}}{aH^2} = -1 - \frac{\dot{H}}{H^2}. \quad (50)$$

This transition can be identified by sign changing of the deceleration parameter. For example, the late-time quintessence era occurs within the interval $-1 \leq q < 0$ at redshift $z \ll 1$ [100]. The deceleration eras are described by $q > 0$ interval. The condition $q = 0$ is a point where the sign changing occurs and the deceleration-acceleration transition realizes for some transition redshifts z_t. In order to find this transition point we make use of the two Friedmann eqs. (10), (11) which can be written as follows,

$$H^2 = H_0^2[\frac{\Omega_0(1+z)^3}{F_R} + \Omega_{eff}],$$
$$\dot{H} = H_0^2(\frac{-3\Omega_0(1+z)^3}{2F_R}) \quad (51)$$

where we have taken non-relativistic dust matter $w = 0$ and vacuum energy or flat ΛCDM model $w_{eff} = -1$ in the calculations. $H_0 = 1.37187 \times 10^{-33}$ eV is the present value of Hubble parameter, and we have introduced density parameters, respectively, for matter and effective one as $\Omega_m(z) = \frac{\Omega_0(1+z)^3}{F_R}$ and $\Omega_{eff} = \frac{8\pi G_D \rho_{eff}}{3H_0^2}$, with $\Omega_0 \equiv \frac{\rho_0}{\rho_{cr}}$ and $\rho_{cr} \equiv \frac{3H_0^2}{8\pi G_D}$. Besides, the scale factor in terms of redshift is given by $a = (1+z)^{-1}$, with the present value $a_0 = 1$. Using normalization condition from eq. (10) $(\Omega_m + \Omega_{eff} = 1)$ we obtain the deceleration parameter as follows,

$$q = -1 + \frac{3\Omega_0(1+z)^3}{2[\Omega_0((1+z)^3 - 1) + F_R]}, \quad (52)$$

where recall that $\rho_m = \frac{\rho}{F_R}$. As one can see, the deceleration parameter depends on current matter density parameter, redshift and higher dimensional $F(R)$ model. Especially, the $F(R)$-dependence offers an advantage when examining the acceleration-deceleration transition of the universe. For example, in Fig. 1, we use the value $\Omega_0 \sim 0.3$ and higher dimensional Einstein term, $F(R) = R$ to show deceleration-acceleration transition, and we find that this transition takes place at the redshift $z_t = 0.67$, which is in good agreement with Ref. [101], and assort with Ref. [100] when using $H(z)$ data only. If z is larger than 4 [102], the radiation era becomes important. Moreover, the limit $z \to \infty$ [104] indicates the radiation era $q = 0.5$ which is a lower boundary of the mechanism for the acceleration-deceleration connection. It can be also seen this boundary point in view of the thermodynamical dark energy parametrization [103].

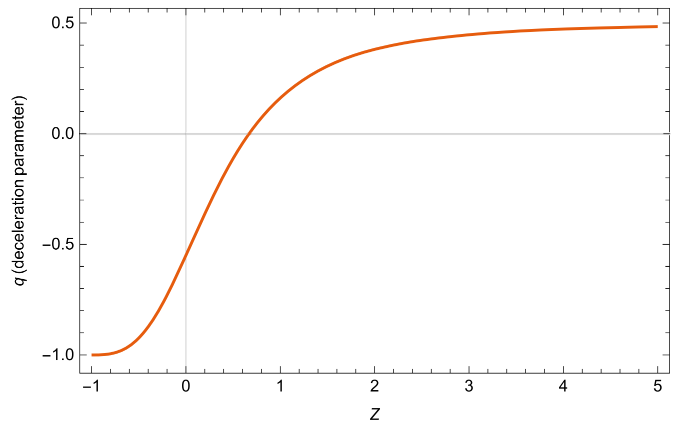

Fig. 1. Deceleration parameter of the mechanism as a function of redshift. The deceleration parameter of the mechanism changes sign ($q = 0$) at the transition redshift $z_t = 0.67$. We use the value $\Omega_0 \sim 0.3$ and Higher dimensional Einstein term, $F(R) = R$ to show deceleration-acceleration redshift transition.

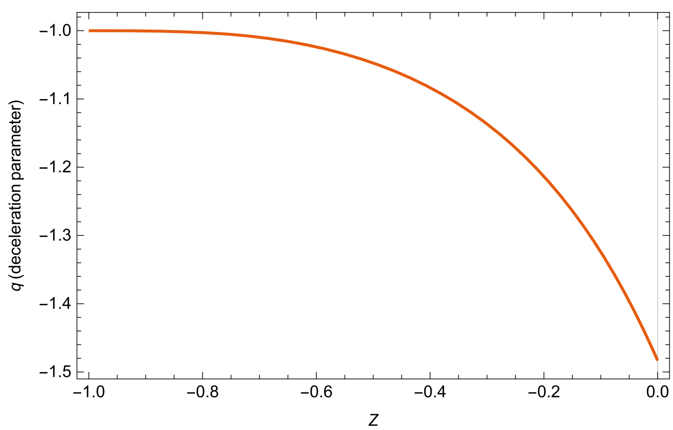

Fig. 2. The deceleration parameter versus redshift for the phantom era.

On the other hand, for βR^{-1} gravity (where $R = 169,38 \times 10^{-66}$ eV^2 from (30)), using $\beta \sim 1080.04 \times 10^{-132}$ eV^4, we find the late-time de Sitter phase occurs at the redshift $z = -1$, and $F(R)$ function behaves as a cosmological constant as $F(R) \sim 1.13\Lambda$ (see below eq. (45)). Moreover, the phantom era is obtained at $-1 \leq z \leq 0$, where $F_R \simeq -0.931 \times 10^{-132} \times (t_s - t)^{-4}$ with $(t_s - t) \simeq 10^{-33}$ order. Especially, at present $z = 0$ we find the EoS parameter as $w \sim -1.32$ which coincides with obtained range $-1.66 < w < -1.22$ for the phantom era corresponding to the range $1 < h < 3$ (see below eq. (49)). In Fig. 2, the mechanism fits the Type Ia supernova data, allowing for the phantom range $w = -1.31 \pm^{0.22}_{0.28}$ [105–109].

5. Closing remarks

In this study, using higher dimensional $F(R)$ gravity, and higher dimensional spatially flat-FRW metric with the scale factor relationship $b = a^{-n}$ [86], we examined historical universe from the early-time inflation period to the late-time phantom era in view of the entropic analysis. We have considered the condition $p_d = -\rho$ to hold the conservation of the matter in all the eras of the universe. This condition provides the point $n = -1$ in the both the inflation and the late-time de Sitter phase. Furthermore, the same condition produced the range $n > 0$ in both the deceleration and the phantom eras which show the same configuration, i.e. four-dimensional spherical and symmetrical spacetime. In the late-time quintessential phase it is only obtained the range $-1 < n < 0$ showing a small deformation on the metric. In summary, we set the following items.

1. The super-inflation scenario has been considered for the inflation era, where the two cases arise. In the first case, the universe, which is in a vacuum state at the beginning of the inflation, has zero entropy with $d = 0$, that causing a singularity on scale factor a. This requires that de Sitter vacua turns into another type of dark energy, i.e. the quintessence dark energy. The second case is a case that the universe has $d + 4$ dimensional spherically symmetric form. But here we do not know the number of dimensions d. However we have obtained a 10 dimensional spherically symmetric spacetime model in the process from the quintessential dark energy phase to the matter creation phase.
2. The deceleration era has two phases, the radiation and dust phases. In the radiation phase, 5 dimensional Kaluza-Klein space-time is obtained. The analogous form of the current structure of the universe appears in the dust phase, where the extra 6 dimensions are compactified into the 4 visible space-time dimensions.
3. In small curvatures of the spacetime, the $1/R$ term becomes dominated and the universe evolves into a new expansion phase after the deceleration era. The range $-1 < n < 0$, which notifies a small deformation on the metric, causes the accelerated expansion of the universe. We conclude that according to the mechanism, in a recent history, the universe was in a state of 10 dimensional spacetime, and this may have triggered the late-time accelerated expansion.

4. While it is obtained 10 dimensional spherically symmetrical model of spacetime in de Sitter phase, the universe enters a phantom era and so gains again 4 dimensional symmetric structure as in dust phase. This indicates that we may now be living in the phantom era. As in the dust phase, in the phantom era of the universe we observe that the extra dimensions are compactified on the standard four spacetime dimensions. But, the universe accelerates in this era.

While the early-time vacuum and the quintessence and also the late-time vacuum and the quintessence (with a small metric deformation) show the same dynamics, the dust phase in deceleration and the phantom phase in the acceleration show similar structure. Therefore, we can talk about a symmetry between the term R^2 and $1/R$. It can be discussed whether the higher dimensional $F(R)$ dynamics will enter the swampland conjuncture as an effective field theory. The swampland conjecture locates vast position of a landscape by drawing the borders of low-energy effective theories, in particular, coming from the string theory with its proposed criteria. At this point, an effective field theory defined out of the string landscape but in swampland is incompatible with quantum gravity. In this study, in the quasi-de Sitter vacuum phase for the early universe, we did not obtain 10-dimensional spacetime in string theory or 11-dimensional spacetime in M-theory. On the contrary, it is obtained a universe model with symmetrical $4+d$ dimensional multiple possibilities. The quintessence field has a critical role, at this point, in uncovering plausible inflation in these multiple possibilities and incorporating it into the string landscape. Because, we know that R^2-term has the string duality symmetry due to Noether symmetries [95,96]. Also, de Sitter vacuum field, which has a positive cosmological constant in high energy regimes, is not placed in the landscape. The instability of the de Sitter phase can be viewed from a different perspective [110,111]. Dark energy, described within the framework of the quintessence field in string theory, shows that it can be determined that the current universe is to be in this form. In fact, the result obtained in the framework of entropic analysis is that inflation does not end with the de Sitter phase, and a reasonable inflation from D-dimensional spacetime only emerges from the quintessence field as 10 spacetime dimensions. On the other hand, In the context of the scale factor duality-invariant approach [112] it can be seen the Noether symmetry approach as third criterion [95] in positioning suitable string-$F(R)$ models in the swampland as an effective mechanism in the string landscape. Just as the term R^2 is a paradigm that can be included in the landscape within the framework of the string duality, the late-time behavior of the Universe may be invariant under the string duality. We have shown that both terms exhibit the same configuration for the dark energy regimes of the universe in the entropic framework. If so, this means that entropic analysis can be a separate swampland criterion. Of course, it is needed a deep analyze related to this issue and we will investigate this phenomenological issue in a future another study.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

References

[1] S.W. Allen, et al., Mon. Not. R. Astron. Soc. 353 (2004) 457.
[2] S. Perlmutter, et al., Nature 391 (1998) 51;
S. Perlmutter, et al., Astrophys. J. 517 (1999) 565.
[3] A.G. Riess, et al., Supernova Search Team, Astron. J. 116 (1998) 1009.
[4] C.L. Bennett, et al., Astrophys. J. Suppl. Ser. 148 (2003) 1.
[5] D.N. Spergel, et al., Astrophys. J. Suppl. Ser. 148 (2003) 175.
[6] S.W. Allen, et al., Mon. Not. R. Astron. Soc. 353 (2004) 457.
[7] P.A.R. Ade, et al., Astron. Astrophys. 571 (2014) 16.
[8] P.A.R. Ade, et al., Astron. Astrophys. 594 (2016) 13.
[9] N. Aghanim, et al., arXiv:1807.06209.
[10] E. Palti, arXiv:1903.06239 [hep-th], 2019.
[11] M.C. David Marsh, Phys. Lett. B 789 (2019) 639.
[12] G. Obied, H. Ooguri, L. Spodyneiko, C. Vafa, arXiv:1806.08362 [hep-th], 2018.
[13] C. Vafa, The string landscape and the Swampland, arXiv:hep-th/0509212.
[14] P. Agrawal, G. Obied, P.J. Steinhardt, C. Vafa, Phys. Lett. B 784 (2018) 271.
[15] A.I. Keskin, I. Acikgoz, Mod. Phys. Lett. A 32 (2017) 1750182.
[16] V.K. Oikonomou, Ann. Phys. 432 (2021) 168576.
[17] V.K. Oikonomou, Class. Quantum Gravity 38 (19) (2021) 195025.
[18] V.K. Oikonomou, F.P. Fronimos, Nucl. Phys. B 971 (2021) 115522.
[19] V.K. Oikonomou, Phys. Rev. D 103 (12) (2021) 124028.
[20] S. Nojiri, S.D. Odintsov, Phys. Rep. 505 (2011) 59.
[21] R. Kerner, Gen. Relativ. Gravit. 14 (1982) 453.
[22] J.D. Barrow, A. Ottewi, J. Phys. A 16 (1983) 2757.
[23] V. Faraoni, Phys. Rev. D 74 (2006) 023520.
[24] S. Nojiri, S.D. Odintsov, Phys. Lett. B 576 (2003) 5.
[25] S. Capozziello, Int. J. Mod. Phys. D 11 (2002) 483.
[26] Sean M. Carroll, et al., Phys. Rev. D 71 (2005) 063513.
[27] S. Nojiri, S.D. Odintsov, Phys. Rev. D 74 (2006) 086005.
[28] S. Capozziello, S. Nojiri, S.D. Odintsov, Phys. Lett. B 634 (2006) 93.
[29] S. Nojiri, S.D. Odintsov, D. Saez-Gomes, Phys. Lett. B 681 (2009) 74.
[30] S. Nojiri, S.D. Odintsov, Mod. Phys. Lett. A 19 (2004) 627.
[31] M.C.B. Abdalla, S. Nojiri, S.D. Odintsov, Class. Quantum Gravity 22 (2005) L35.
[32] A. Baykal, O. Delice, Class. Quantum Gravity 28 (2011) 015014.
[33] S. Nojiri, S.D. Odintsov, V.K. Oikonomou, Phys. Rep. 692 (2017) 1–104.
[34] S.D. Odintsov, V.K. Oikonomou, Phys. Rev. D 92 (2) (2015) 024016.

[35] V.K. Oikonomou, Phys. Rev. D 92 (12) (2015) 124027.
[36] S. Bahamonde, S.D. Odintsov, V.K. Oikonomou, P.V. Tretyakov, Phys. Lett. B 766 (2017) 225.
[37] A.I. Keskin, I. Acikgoz, Astrophys. Space Sci. 361 (2016) 391.
[38] S. Capozziello, M. De Laurentis, Phys. Rep. 509 (2011) 167.
[39] G.G.L. Nashed, W. El Hanafy, S.D. Odintsov, V.K. Oikonomou, Int. J. Mod. Phys. D 29 (2020) 2050090.
[40] S. Nojiri, S.D. Odintsov, V.K. Oikonomou, Phys. Dark Universe 29 (2020) 100602.
[41] S.D. Odintsov, V.K. Oikonomou, S. Banerjee, Nucl. Phys. B 938 (2019) 935.
[42] A. Guth, Phys. Rev. D 23 (1981) 347.
[43] A.A. Starobinsky, Phys. Lett. B 91 (1980) 99.
[44] M. Salti, O. Aydogdu, H. Yanar, K. Sogut, Ann. Phys. 390 (2018) 131.
[45] A.I. Keskin, M. Salti, Phys. Lett. B 791 (2019) 80.
[46] K. Andrew, B. Bolen, Chad A. Middleton, Phys. Rev. D 39 (2007) 2061.
[47] M. Korunur, Class. Quantum Gravity 38 (2021) 075004.
[48] T. Jacobson, Phys. Rev. Lett. 75 (1995) 1260.
[49] C. Eling, R. Guedens, T. Jacobson, Phys. Rev. Lett. 96 (2006) 121301.
[50] M. Akbar, R.G. Cai, Phys. Lett. B 635 (2006) 7.
[51] M. Akbar, R.G. Cai, Phys. Lett. B 648 (2007) 243.
[52] T. Padmanabhan, Phys. Rep. 406 (2005) 49.
[53] T. Padmanabhan, Rep. Prog. Phys. 73 (2010) 046901.
[54] T. Padmanabhan, Class. Quantum Gravity 19 (2002) 5387.
[55] T. Padmanabhan, Int. J. Mod. Phys. D 15 (2006) 1659.
[56] U. Yeter, K. Sogut, M. Salti, Found. Phys. 51 (2021) 19.
[57] O. Siginc, et al., Mod. Phys. Lett. A 33 (2018) 1850137.
[58] P. Rudra, B. Pourhassan, Phys. Dark Universe 33 (2021) 100849.
[59] S.A. Hayward, Class. Quantum Gravity 15 (1998) 3147.
[60] S.A. Hayward, S. Mukhoyama, M.C. Ashworth, Phys. Lett. 256 (1999) 347.
[61] S.A. Hayward, Phys. Rev. Lett. 93 (2004) 251101.
[62] R.G. Cai, L.M. Cao, Phys. Rev. D 75 (2007) 064008.
[63] R.G. Cai, S.P. Kim, J. High Energy Phys. 0502 (2005) 050.
[64] R.G. Cai, Y.S. Myung, Phys. Rev. D 67 (2003) 124021.
[65] R.G. Cai, L.M. Cao, Nucl. Phys. B 785 (2007) 135.
[66] R.G. Cai, L.M. Cao, Y.P. Hu, J. High Energy Phys. 0808 (2008) 090.
[67] S. Mitra, S. Saha, S. Chakraborty, Phys. Lett. B 734 (2014) 173.
[68] S. Mitra, S. Saha, S. Chakraborty, Ann. Phys. 355 (2015) 1.
[69] A. Sheykhi, B. Wang, R.G. Cai, Nucl. Phys. B 779 (2007) 1.
[70] A. Sheykhi, B. Wang, R.G. Cai, Phys. Rev. D 76 (2007) 023515.
[71] A. Sheykhi, Phys. Lett. B 785 (2018) 118.
[72] E.N. Saridakis, S. Basilakos, Eur. Phys. J. C 81 (2021) 644.
[73] A.G. Shalaby, V.K. Oikonomou, G.G.L. Nashed, Symmetry 13 (2021) 75, https://doi.org/10.3390/sym13010075.
[74] S. Nojiri, et al., Nucl. Phys. B 950 (2020) 114850.
[75] E.N. Saridakis, K. Bamba, R. Myrzakulov, Fotios K. Anagnostopoulos, J. Cosmol. Astropart. Phys. 1812 (2018) 012.
[76] A.M. zta, E. Dil, Phys. Dark Universe 31 (2021) 100788.
[77] M. Akbar, R.G. Cai, Phys. Rev. D 75 (2007) 084003.
[78] A.V. Frolov, L. Kofman, J. Cosmol. Astropart. Phys. 0305 (2003) 009.
[79] U.K. Danielsson, Phys. Rev. D 71 (2005) 023516.
[80] G. Calcagni, J. High Energy Phys. 0509 (2005) 060.
[81] B. Wang, E. Abdalla, R.K. Su, Phys. Lett. B 503 (2001) 394.
[82] B. Wang, E. Abdalla, R.K. Su, Mod. Phys. Lett. A 17 (2002) 23.
[83] S. Capozziello, V.F. Cardone, S. Carloni, A. Troisi, Int. J. Mod. Phys. D 12 (2003) 1969.
[84] M. Ibe, M. Yamazaki, T.T. Yanagida, arXiv:1811.04664 [hep-th].
[85] T.P. Sotiriou, V. Varaoni, Rev. Mod. Phys. 82 (2010) 451.
[86] N. Mohammedi, Phys. Rev. D 65 (2002) 104018.
[87] D. Bak, S.J. Rey, Class. Quantum Gravity 17 (2000) L83.
[88] R.M. Wald, Phys. Rev. D 48 (1993) 3427.
[89] Sean M. Carroll, V. Duvvuri, M. Trodden, Michael S. Turner, Phys. Rev. D 70 (2004) 043528.
[90] A.I. Keskin, Astrophys. Space Sci. 362 (2017) 50.
[91] A.I. Keskin, Int. J. Mod. Phys. D 27 (2018) 1850112.
[92] A.I. Keskin, Eur. Phys. J. C 78 (2018) 705.
[93] N. Kaloper, L. Sorbo, Phys. Rev. D 79 (2009) 043528.
[94] S. Capozziello, S.J. Gabriele Gionti, D. Vernieri, J. Cosmol. Astropart. Phys. 1601 (2016) 015.
[95] M. Benetti, S. Capozziello, L.L. Graef, Phys. Rev. D 100 (2019) 084013.
[96] S. Capozziello, R. De Ritis, C. Rubano, P. Scudellaro, Riv. Nuovo Cimento 19 (4) (1996) 1.
[97] R.B. Mann, D.E. Vincent, Phys. Lett. A 107 (1985) 75.
[98] Y. Okada, Nucl. Phys. B 264 (1986) 197.
[99] J.M. Overduin, P.S. Wesson, Phys. Rep. 283 (1997) 303.
[100] S. Capozziello, O. Farooq, O. Luongo, B. Ratra, Phys. Rev. D 90 (2014) 044016.
[101] O. Farooq, F.R. Madiyar, S. Crandall, B. Ratra, Astrophys. J. 835 (2017) 26.
[102] S. Capozziello, P. Martin-Moruno, C. Rubano, Phys. Lett. B 664 (2008) 12.
[103] S. Capozziello, R. D'Agostino, O. Luongo, Phys. Dark Universe 36 (2022) 101045.
[104] S.H. Shekh, Phys. Dark Universe 33 (2021) 100850.
[105] S. Hannestad, E. Mortsell, Phys. Rev. D 66 (2002) 063508.
[106] A. Melchiorri, L. Mersini, C.J. Odman, M. Trodden, arXiv:astro-ph/0211522.
[107] S.M. Carroll, M. Hoffman, M. Trodden, arXiv:astro-ph/0301273.
[108] P. Singh, M. Sami, N. Dadhich, Phys. Rev. D 68 (2003) 023522.
[109] A.G. Riess, et al., Astrophys. J. 607 (2004) 665.
[110] D. Andriot, C. Roupec, Fortschr. Phys. 67 (2019) 1800105.
[111] A. Ashoorioon, Phys. Lett. B 790 (2019) 568.
[112] S. Capozziello, R. de Ritis, Int. J. Mod. Phys. D 2 (1993) 367.